Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Student number\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Assignment 3**

A bar is loaded by its own weight as shown in the figure. Determine the equilibrium equation in terms of the dimensionless displacement  with the large deformation theory. Without external loading, area of the cross-section, length of the bar, and density of the material are *A*, *L*, and , respectively. Young’s modulus of the material is *C*. Also find the displacement according to the linear theory by simplifying the equilibrium equation with the assumption .

*g*

*X*

*Z*



*x*

*y*

1

2

1

**Solution template**

Virtual work densities of the bar model

,



are based on the Green-Lagrange strain definition, which works also when rotations/displacements are large. The expressions depend on all displacement components, material property is denoted by  (kind of Young’s modulus), and the superscript in the cross-sectional area  (and in other quantities) refers to the initial geometry where strain and stress vanish.

The non-zero displacement component of the structure is the vertical displacement of node 2. Linear approximations to the displacement components in terms of the displacement/rotation components of the structural system are

 and    and .

In terms of the dimensionless displacement , virtual work densities simplify to

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.

Virtual work expressionsare integrals of the densities over the domain occupied by the element

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Principle of virtual work and the fundamental lemma of variation calculus imply that

 in which . 🡸

Assuming that , only the linear part in matters and the equilibrium equation simplifies to

  . 🡸